QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

QUADRATURE OF THE CIRCLE.

By EDWARD J. GOODWIN, Solitude, Indiana.

Published by the request of the author.

A circular area is equal to the square on a line equal to the quadrant of the circumference; and the area of a square is equal to the area of the circle whose circumference is equal to the perimeter of the square.

To quadrate the circle is to find the side of a square whose perimeter equals that of the given circle; rectification of the circle requires to find a right line equal to the circumference of the given circle. The square on a line equal to the arc of 90° fulfills both of the said requirements.

It is impossible to quadrate the circle by taking the diameter as the linear unit, because the square root of the product of the diameter by the quadrant of the circumference produces the side of a square which equals 9 when the quadrant equals 8.

It is not mathematically consistent that it should take the side of a square whose perimeter equals that of a greater circle to measure the space contained within the limits of a less circle.

Were this true, it would require a piece of tire iron 18 feet to bind a wagon wheel 16 feet in circumference.

This new measure of the circle has happily brought to light the ratio of the chord and arc of 90°, which is as 7:8; and also the ratio of the diagonal and one side of a square, which is as 10:7. These two ratios show the numerical relation of diameter to circumference to be as $\frac{5}{4}:4$.

Authorities will please note that while the finite ratio ($\frac{5}{4}:4$) represents the area of the circle to be more than the orthodox ratio, yet the ratio (3.1416) represents the area of a circle whose circumference equals 4 two % greater than the finite ratio ($\frac{5}{4}:4$), as will be seen by comparing the terms of their respective proportions, stated as follows: $1:3.20::1.25:4$, $1:3.1416::1.2732:4$.

It will be observed that the product of the extremes is equal to the product of the means in the first statement, while they fail to agree in the second proportion. Furthermore, the square on a line equal to the arc of 90° shows very clearly that the ratio of the circle is the same in principle as that of the square. For example, if we multiply the perimeter of a square (the sum of its sides) by $\frac{1}{4}$ of one side the product equals the sum of two sides by $\frac{1}{2}$ of one side, which equals the square on one side.

Again, the number required to express the units of length in $\frac{1}{4}$ of a right line, is the square root of the number representing the squares of the linear unit bounded by it in the form of a square whose ratio is as 1:4.

These properties of the ratio of the square apply to the circle without an exception, as is further sustained by the following formula to express the numerical measure of both *circle* and *square.*

Let C represent the circumference of a circle whose quadrant is *unity*, Q $\frac{1}{2}$ the quadrant, and CQ^2 will apply to the numerical measure of a circle and a square.

We are now able to get the true and finite dimensions of a circle by the exact ratio $\frac{5}{4}:4$, and have simply to divide the circumference by 4 and square the quotient to compute the area.

"An Unreasonable Rule." By SETH PRATT, C. E., Tecumseh, Nebraska.

Professor Philbrick is mistaken as the following Table of convergences of two meridians $69\frac{1}{6}$ miles apart at the equator and extending to the pole for parallels 1° of difference will show. We have R: diff. of cosines :: $69\frac{1}{6}$: convergences.

The object of the Rule is to conform to the law, which requires that "the east and west boundaries of the townships shall conform to the true meridians, and that north and south boundaries shall run on paralles of latitude and, further, that the townships shall be 6 miles square as "nearly as may be."

It is readily seen that the distances between any two meridians diminishes as the co-sines of the latitudes, and that corrections should be applied at less distances from any given parallel extending toward the pole, than in a contrary direction. Hence the Rule is "Reasonable."

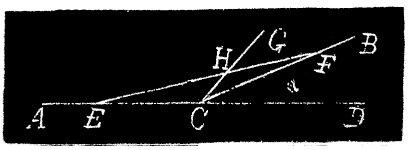
Lat.	Convergence.	Lat.	Convergence.
0° 1°	0.845ch.	50° 51°	73.786
10° 11°	17.323	60° 61°	84.054
20° 21°	33.819	70° 71°	91.034
30° 31°	49.010	80° 81°	95.249
40° 41°	62.619	89° 90°	96.570

LOBATCHEWSKY'S TRIANGLE. By Professor JOHN N. LYLE, Ph. D, Westminster College, Fulton. Missouri.

"There may be a triangle whose angle sum differs from a straight angle" (two right angles) "by less than any given finite angle however small."

If there may be such a triange the possibility of its construction must be granted. Then let us construct this triangle and call it ECH.

According ho hypothesis this triangle ECH is "a triangle" whose angle sum is equal to two right angles minus the angle a which is "less than any given finite angle however small."



Extend the line EC both ways to

A and D, and construct the angle ECB equal to the angle sum of the triangle ECH, or two right angles$-a$. Then, the supplement of ECB is DCB $=a$. The individual angle DCB is the difference between the finite angle ECF less than two right angles and two right angles and is, therefore, finite.

But the angle DCB by hypothesis is less than any finite angle. Hence, contradictory marks are attributed to the supplementary angle DCB or a, which is absurd. Since the conclusion is absurd, the hypothesis from which it is deduced must be unsound.

"REMARKS ON DIVISION." By WILLIAM F. BRADBURY, Cambridge, Massachusetts.

I take issue with Mr. Ellwood in many of his statements in this article.

1st "If a given product is $20, and the multiplier 4, we cannot by mere subtraction, find the multiplicand." But we can. Try some number say 4 and subtract thus: $20-4=16$: $16-4=12$: $12-4=8$: $8-4=4$. Now we have subtracted 4 times but have a remainder. Try 5 thus: $20-5=15$: $15-5=10$: $10-5=5$: $5-5=0$. Thus we find a number (5) which subtracted four times (as above) becomes 0. Therefore $5 is the multiplicand.

Therefore *all* that he says about *abstract* and *concrete* falls to the ground.

2nd. So too $\frac{1}{4}$ of $\frac{8}{9}$ has been called by mathematicians from time immemorial a *compound fraction*. Therefore, this *is the name of it*. And further it is an example in multiplication of fractions. $\frac{1}{4}$ of $\frac{8}{9}=\frac{1}{4}\times\frac{8}{9}$. Now this sign, $\times$, is the sign of multiplication. It is *not* an example in division of fractions. It is an old story that multiplying by a number less than a unit gives a product that is less than the multiplicand. Because multiplying a number by $\frac{1}{4}$ is the same as dividing that number by 4. I do not call multiplying by $\frac{1}{4}$ an example in division. It is a mere quibble in words. What has been so named may as well keep its name.

3rd. So $\dfrac{4}{\frac{2}{3}}$ *is* a complex fraction, because *this is its name*. A fraction may well be defined as an indicated division. Thus $\frac{8}{9}$ may be read eight-ninths, or eight divided by nine. I am perfectly willing to leave $\dfrac{4}{\frac{2}{3}}$ and other complex fractions "as special gifts from high."

EDITORIALS

Professor Leonard E. Dickson goes to Chicago University as Fellow in Pure Mathematics, having resigned a Shattuck scholarship at Harvard to accept the Fellowship in Chicago University.

All of our subscribers who have not received all of the 7 Nos. of the MONTHLY will please write to the publishers at once, and tell them which Nos. you failed to receive.